INTRODUCTORY ECONOMETRICS

Lesson 2a

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2 The Linear Regression Model (I). Specification and Estimation.

2.1 Specification of the General Linear Regression Model (GLRM).

Specification of the GLRM (1)

- **Objective:** Quantifying the relationship between:
  - a variable Y and
  - a set of K explanatory variables $X_1, X_2, \ldots, X_K$,
  - by means of a linear model.
- **Starting point:**
  - a linear model: $Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_K X_K + u$,
  - a data sample of size T: $Y_t, X_{1t}, X_{2t}, \ldots, X_{Kt},\ t = 1 \ldots T$, where

$$Y_t = t\text{-th obs of } Y,$$

$$X_{kt} = t\text{-th obs of } X_k,\ k = 1, 2 \ldots K.$$

Specification of the GLRM (2)

- GLRM:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \cdots + \beta_K X_{Kt} + u_t,\ t = 1, 2 \ldots T,$$

whose elements are (recall):
  - Y: dependent variable,
  - $X_k,\ k = 1 \ldots K$: explanatory variables,
  - β_0: intercept,
    $\beta_k,\ k = 1 \ldots K$: coefficients (parameters to be estimated),
  - u: (non-observable random) error or disturbance, that allows for:
    - variables not included in the model,
    - random behaviour of economic agents,
    - measurement errors, etc.

The GLRM in observation form

The model

$$Y_t = \beta_0 + \beta_1 X_{1t} + \cdots + \beta_K X_{Kt} + u_t, \; t = 1, 2 \ldots T,$$

implies for each observation:

$$
\begin{aligned}
Y_1 &= \beta_0 + \beta_1 X_{11} + \beta_2 X_{21} + \cdots + \beta_K X_{K1} + u_1 \\
Y_2 &= \beta_0 + \beta_1 X_{12} + \beta_2 X_{22} + \cdots + \beta_K X_{K2} + u_2 \\
&\cdots\cdots \\
Y_t &= \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_K X_{Kt} + u_t \\
&\cdots\cdots \\
Y_T &= \beta_0 + \beta_1 X_{1T} + \beta_2 X_{2T} + \cdots + \beta_K X_{KT} + u_T
\end{aligned}
$$

The GLRM in matrix form (1)

or else in matrix form:

$$
\begin{bmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_t \\ \cdots \\ Y_T \end{bmatrix}
=
\begin{bmatrix}
\beta_0 + \beta_1 X_{11} + \beta_2 X_{21} + \cdots + \beta_K X_{K1} \\
\beta_0 + \beta_1 X_{12} + \beta_2 X_{22} + \cdots + \beta_K X_{K2} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_K X_{Kt} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\beta_0 + \beta_1 X_{1T} + \beta_2 X_{2T} + \cdots + \beta_K X_{KT}
\end{bmatrix}
+
\begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_t \\ \cdots \\ u_T \end{bmatrix}
$$

The GLRM in matrix form (2)

- that is:

$$
\underset{(T\times 1)}{\underset{Y}{\begin{bmatrix} Y_1 \\ Y_2 \\ \cdots \\ Y_t \\ \cdots \\ Y_T \end{bmatrix}}}
=
\underset{(T\times K{+}1)}{\underset{X}{\begin{bmatrix}
1 & X_{11} & X_{21} & \ldots & X_{K1} \\
1 & X_{12} & X_{22} & \ldots & X_{K2} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
1 & X_{1t} & X_{2t} & \ldots & X_{Kt} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
1 & X_{1T} & X_{2T} & \ldots & X_{KT}
\end{bmatrix}}}
\underset{(K{+}1\times 1)}{\underset{\beta}{\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_K \end{pmatrix}}}
+
\underset{(T\times 1)}{\underset{u}{\begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_t \\ \cdots \\ u_T \end{bmatrix}}}
$$

-

$$\boxed{Y = X\beta + u.}$$

↰

2.2 Basic (Classical) Assumptions. Interpretation.

Basic Assumptions of the GLRM (1)

1. Assumptions about the relationship:
   - Model is correctly specified:

$$X_k \text{ explains } Y \Leftrightarrow X_k \in \text{model.}$$

2. Assumptions about the parameters:
   - they are constant throughout the sample,
   - they appear linearly (*i.e.* a constant plus coefficients)
     - $Y_t = \beta_0 + \beta_1 X_t + u_t$
   - Note: but vars $Y, X_1, X_2, \ldots$ may be transformations:
     - $Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_t^2 + \beta_3 \dfrac{1}{X_t} + u_t$
     - $Y_t = A X_{1t}^{\beta_1} X_{2t}^{\beta_2} e^{u_t}$ (Why?)
     - and this? $Y_t = \beta_0 + \beta_1 \dfrac{1}{X_t - \beta_2} + u_t$
     - and these other? $\ln Y_t = \beta_0 X_t^{\beta_1} u_t; \quad Y_t = \beta_0 X_t^{\beta_1} + u_t$
       $Y_t = \beta_1 X_{1t} + \beta_2 X_{1t} X_{2t} + u_t; \quad Y_t = \beta_0 + \beta_1 X_{1t}^{X_{2t}} + u_t$

Basic Assumptions of the GLRM (2)

3. Assumptions about the explanatory variables:
   (a) $X_1, \ldots, X_K$, are quantitative and fixed (*i.e.* not random).
   (b) $X_1, \ldots, X_K$, are linearly independent:
   - $\nexists X_k | X_k =$ lin. comb. of others (Why?)

- Examples of not valid cases:
  - $Y_t = \beta_0 + \beta_1 X_t + \beta_2 (2X_t + 3) + u_t$
  - $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 (X_{1t} + X_{2t}) + u_t$
- Examples of valid cases:
  - $Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_t^2 + u_t$
  - $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{1t} X_{2t} + u_t$

Basic Assumptions of the GLRM (3)

4. Assumptions about the disturbance term:
   (a) Zero mean:
   $\mathrm{E}(u_t) = 0 \quad \forall t$ (isn't obvious?).
   (b) Homoscedastic:
   $\mathrm{Var}(u_t) = \mathrm{E}(u_t^2) = \sigma_u^2 \ (= \sigma^2)$ const ($\forall t$).
   (c) Serially uncorrelated:
   $\mathrm{Cov}(u_t, u_s) = \mathrm{E}(u_t u_s) = 0 \quad \forall t \neq s.$
   (d) Normally distributed$^{(*)}$:
   $u_t \sim \mathcal{N} \quad \forall t.$ (* added)

- Assumptions 4a–4d jointly:

$$u_t \sim \text{iid}\, \mathcal{N}(0, \sigma_u^2)$$

Basic Assumptions in matrix form (1)

- from 4a: Mean Vector:

$$\underset{(T\times 1)}{\mathrm{E}(u)} = \begin{bmatrix} \mathrm{E}(u_1) \\ \mathrm{E}(u_2) \\ \vdots \\ \mathrm{E}(u_T) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 0_T$$

- from 4b and 4c: Covariance Matrix:

$$\underset{(T\times T)}{\mathrm{E}(uu')} = \begin{bmatrix} \mathrm{E}(u_1^2) & \mathrm{E}(u_1 u_2) & \ldots & \mathrm{E}(u_1 u_T) \\ \mathrm{E}(u_2 u_1) & \mathrm{E}(u_2^2) & \ldots & \mathrm{E}(u_2 u_T) \\ \ldots & \ldots & \ldots & \ldots \\ \mathrm{E}(u_T u_1) & \mathrm{E}(u_T u_2) & \ldots & \mathrm{E}(u_T^2) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_u^2 & 0 & \ldots & 0 \\ 0 & \sigma_u^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \sigma_u^2 \end{bmatrix} = \sigma_u^2 I_T$$

Basic Assumptions in matrix form (2)

- more compactly:

$$\underset{(T\times 1)}{u} \sim (\underset{(T\times 1)}{0}, \sigma_u^2 \underset{(T\times T)}{I_T})$$

- plus 4d:

$$\underset{(T\times 1)}{u} \sim \mathcal{N}(\underset{(T\times 1)}{0}, \sigma_u^2 \underset{(T\times T)}{I_T})$$

2.3a Ordinary Least Squares (OLS) in a Single Linear Regression Model (SLRM).

SLRM: the PRF

- With $K=1 \quad \rightsquigarrow \quad Y_t = \beta_0 + \beta_1 X_{1t} + u_t,$

$$\text{(SLRM):} \quad Y_t = \alpha + \beta X_t + u_t. \qquad (1)$$

- Population Regression Function (PRF):
  $\mathsf{E}(u_t) = 0 \quad \rightsquigarrow$ *systematic part* or PRF:

$$\mathsf{E}(Y_t) = \alpha + \beta X_t$$

- Interpretation of the parameters:
  - $\alpha = \mathsf{E}(Y_t | X_t = 0)$: Expected value of Y_t when the explanatory variable is zero.
  - $\beta = \frac{\partial \mathsf{E}(Y_t)}{\partial X_t} \simeq \frac{\Delta \mathsf{E}(Y_t)}{\Delta X_t}$: Increase in (expected) value of Y_t when $X \uparrow$ one unit (*c.p.*).
- Objective: To obtain estimates $\widehat{\alpha}, \widehat{\beta}$ of the unknown parameters α, β in (1).

The Sample Regression Function (SRF)

- $\widehat{\alpha}, \widehat{\beta} \quad \rightsquigarrow$ estimated model or SRF:

$$\widehat{Y}_t = \widehat{\alpha} + \widehat{\beta} X_t$$

- Interpretation of the estimates:
  - $\widehat{\alpha} = (\widehat{Y}_t | X_t = 0)$: Estimated value of Y_t when the explanatory variable is zero.
  - $\widehat{\beta} = \frac{\partial \widehat{Y}_t}{\partial X_t} \simeq \frac{\Delta \widehat{Y}_t}{\Delta X_t}$: Estimated increase in Y_t when $X \uparrow$ one unit (*c.p.*).
- Note difference: an estimator (a formula) vs. an estimate (a number).

Disturbances vs. Residuals

- Disturbances in PRF:

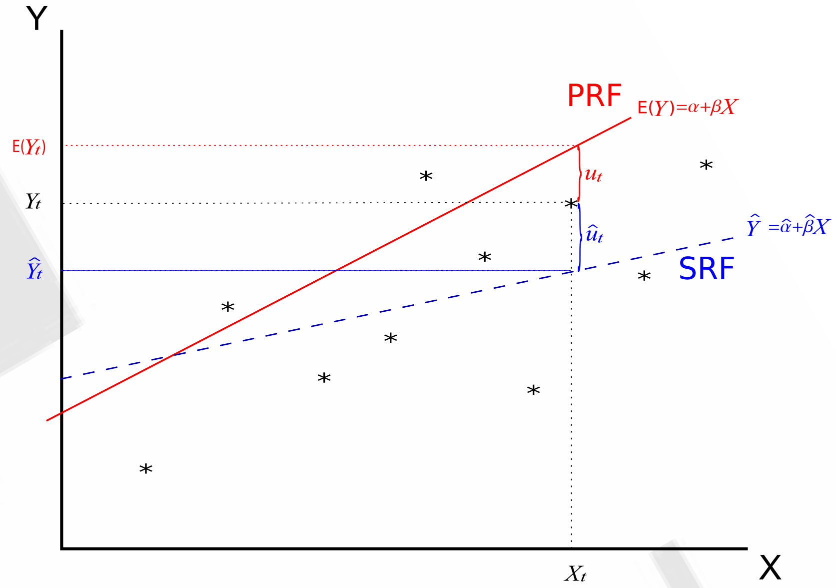
$$u_t = Y_t - \mathrm{E}(Y_t) = Y_t - \alpha - \beta X_t$$

- Residuals in SRF:

$$\widehat{u}_t = Y_t - \widehat{Y}_t = Y_t - \widehat{\alpha} - \widehat{\beta} X_t$$

- Residuals are to the SRF what disturbances are to the PRF.

SLRM: PRF and SRF

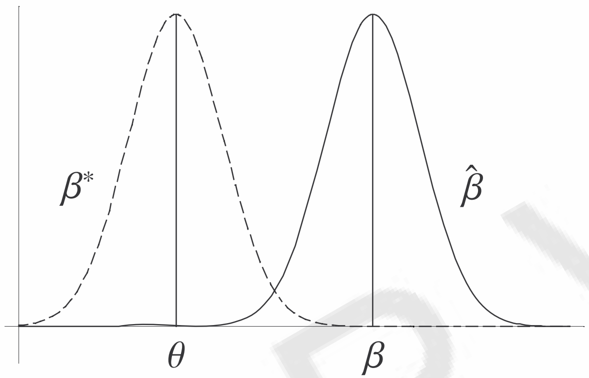


Estimation: Desired Properties (1)

Let $\widehat{\beta}$ be an estimator of $\beta\ldots$

Unbiasedness:

$$\mathrm{E}(\widehat{\beta}) = \beta \quad \Leftrightarrow \quad \widehat{\beta} \text{ unbiased}$$

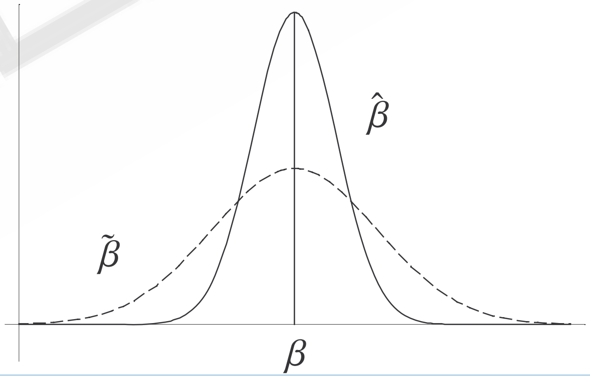


Estimation: Desired Properties (2)

Let $\widehat{\beta}$ and $\widetilde{\beta}$ be unbiased estimators of $\beta\ldots$

Relative efficiency:

$$\mathrm{Var}(\widehat{\beta}) \leq \mathrm{Var}(\widetilde{\beta}) \quad \Leftrightarrow \quad \widehat{\beta} \text{ relatively efficient}$$



Estimation: OLS criteria

$$\text{SLRM: } Y_t = \alpha + \beta X_t + u_t,$$

- apply Least-Squares fit:

$$\min_{\alpha,\beta} \sum_{t=1}^{T} u_t^2 \quad \text{where} \quad u_t = Y_t - \alpha - \beta X_t :$$

- First derivatives:
  - $$\frac{\partial \sum u_t^2}{\partial \alpha} = 2\sum u_t \frac{\partial u_t}{\partial \alpha} = 2\sum u_t(-1)$$
  - $$\frac{\partial \sum u_t^2}{\partial \beta} = 2\sum u_t \frac{\partial u_t}{\partial \beta} = 2\sum u_t(-X_t)$$
- 1st.o.c. (minimum) $\Rightarrow$ first derivatives are zero:
  - $$\sum \widehat{u}_t = \sum (Y_t - \widehat{\alpha} - \widehat{\beta} X_t) = 0$$
  - $$\sum \widehat{u}_t X_t = \sum (Y_t X_t - \widehat{\alpha} X_t - \widehat{\beta} X_t^2) = 0$$

Estimation: Normal equations & LSE of α

- From the above 1st.o.c's:

$$\sum (Y_t - \widehat{\alpha} - \widehat{\beta} X_t) = 0$$
$$\sum (Y_t X_t - \widehat{\alpha} X_t - \widehat{\beta} X_t^2) = 0$$

- we obtain the Normal Equations:

$$\left.\begin{aligned} \sum Y_t &= T\widehat{\alpha} + \widehat{\beta} \sum X_t \\ \sum Y_t X_t &= \widehat{\alpha} \sum X_t + \widehat{\beta} \sum X_t^2 \end{aligned}\right\}$$ 2 equation system with 2 unknowns!!

- Dividing the 1st. normal eq. by T:

$$\frac{1}{T}\sum Y_t = \frac{1}{T} T \widehat{\alpha} + \widehat{\beta} \frac{1}{T} \sum X_t$$

- That is:

$$\boxed{\widehat{\alpha}_{OLS} = \overline{Y} - \widehat{\beta}\,\overline{X}}$$

Estimation: Normal equations & LSE of β

- Substituting $\widehat{\alpha}$ in the 2nd. normal eq.:

$$\sum Y_t X_t = (\overline{Y} - \widehat{\beta}\overline{X}) \sum X_t + \widehat{\beta} \sum X_t^2$$

- . . . dividing by T and gathering terms together:

$$\frac{1}{T}\sum Y_t X_t = (\overline{Y} - \widehat{\beta}\overline{X}) \frac{1}{T}\sum X_t + \widehat{\beta} \frac{1}{T} \sum X_t^2$$
$$\frac{1}{T}\sum Y_t X_t - \overline{Y}\overline{X} = \widehat{\beta}\left(\frac{1}{T}\sum X_t^2 - \overline{X}^2\right)$$

- . . . and solving for the unknown:

$$\widehat{\beta} = \frac{\frac{1}{T}\sum Y_t X_t - \overline{Y}\overline{X}}{\frac{1}{T}\sum X_t^2 - \overline{X}^2} = \frac{\frac{1}{T}\sum y_t x_t}{\frac{1}{T}\sum x_t^2} \quad \begin{bmatrix}\text{Why?}\\ \text{Why?}\end{bmatrix} \quad \longrightarrow$$

- That is:

$$\boxed{\widehat{\beta}_{OLS} = \frac{\sum y_t x_t}{\sum x_t^2} = \frac{\text{Cov}(Y,X)}{\text{Var}(X)}}$$

Recall: variances and covariances?

- variance from original (uncentred) data?

$$\text{Var}(X) = \frac{1}{T}\sum x_t^2 = \frac{1}{T}\sum (X_t - \overline{X})^2$$
$$= \frac{1}{T}\sum X_t^2 + \frac{1}{T}\sum \overline{X}^2 - \frac{2}{T}\overline{X}\sum X_t$$

$$\boxed{\frac{1}{T}\sum x_t^2 = \frac{1}{T}\sum X_t^2 - \overline{X}^2}$$

- covariance from original (uncentred) data?

$$\text{Cov}(Y,X) = \frac{1}{T}\sum x_t y_t = \frac{1}{T}\sum (X_t - \overline{X})(Y_t - \overline{Y})$$
$$= \frac{1}{T}\sum X_t Y_t + \frac{1}{T}\sum \overline{X}\overline{Y} - \frac{1}{T}\overline{Y}\sum X_t - \frac{1}{T}\overline{X}\sum Y_t$$

$$\boxed{\frac{1}{T}\sum x_t y_t = \frac{1}{T}\sum X_t Y_t - \overline{X}\overline{Y}}$$

Numerical example: strawberry prod data

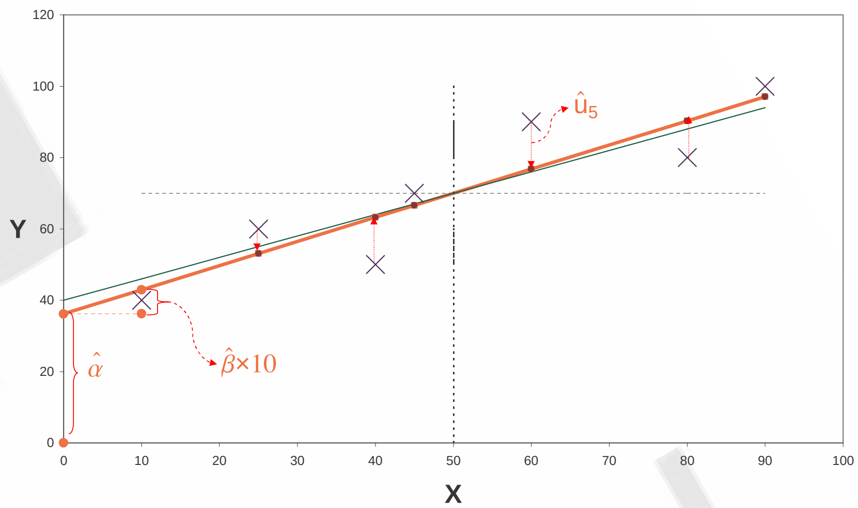
- Data...
- Centred data or "in deviation form"
  (deviations from respective means)...
- Squares and products...

	Y	X	y	x	y^2	x^2	yx
	40	10	-30	-40	900	1600	1200
	60	25	-10	-25	100	625	250
	50	40	-20	-10	400	100	200
	70	45	0	-5	0	25	0
	90	60	20	10	400	100	200
	80	80	10	30	100	900	300
	100	90	30	40	900	1600	1200
Sum					2800	4950	3350
Average	70	50	0	0	400	707.14	478.57

$\widehat{\alpha} = 36.162 \; (= \overline{Y} - \widehat{\beta}\overline{X})$ $\quad\quad$ $\widehat{\beta} = 0.677 \left(= \frac{\text{Cov}(Y,X)}{\text{Var}(X)} \right)$

Can also use formulae based on original data... (Exercise: Try it!!)

Numerical example: strawberry regres plot



2.4a Properties of the Sample Regression Function.

Properties of residuals and SRF (1)

$$\widehat{\beta}_{\text{OLS}} \rightsquigarrow \widehat{\alpha}_{\text{OLS}} \rightsquigarrow \widehat{Y}_t = \widehat{\alpha} + \widehat{\beta}X_t \rightsquigarrow \widehat{u}_t = Y_t - \widehat{Y}_t$$

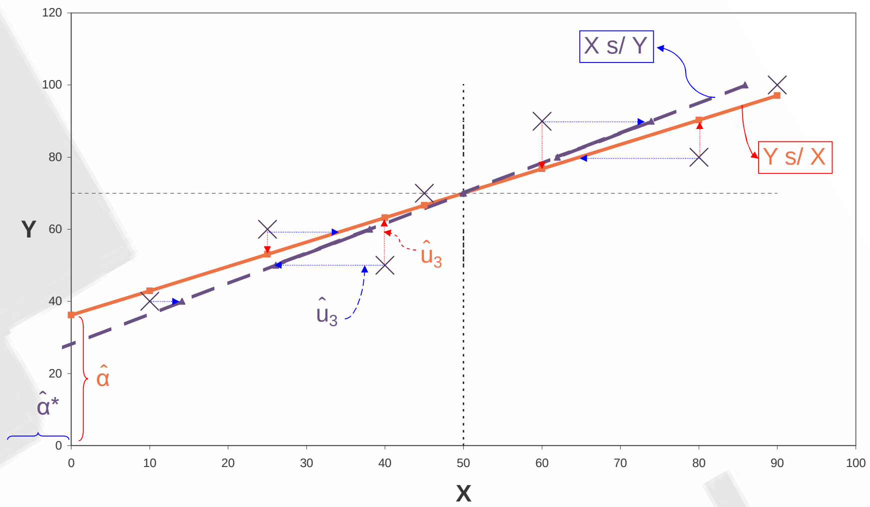
1. residuals add up to zero: $\sum \widehat{u}_t = 0$
   Demo: directly from 1st.o.c. □
2. $\overline{\widehat{Y}} = \overline{Y}$
   Demo: by def.: $\widehat{u}_t = Y_t - \widehat{Y}_t \rightsquigarrow \overline{\widehat{Y}} = \overline{Y} - \overline{\widehat{u}}$,
   but $\overline{\widehat{u}} = \frac{1}{T}\sum \widehat{u}_t = 0$ (from prop 1) $\rightsquigarrow \overline{\widehat{Y}} = \overline{Y}$. □
3. the SRF passes thru the pair of means $(\overline{X}, \overline{Y})$:
   $\overline{Y} = \widehat{\alpha} + \widehat{\beta}\overline{X}$
   Demo: from $\widehat{\alpha} = \overline{Y} - \widehat{\beta}\overline{X}$ (1st. normal eq.) □

Properties of residuals and SRF (2)

4. residuals orthogonal to expl. v. X: $\sum X_t \hat{u}_t = 0$
   Demo: directly from 1st.o.c.
5. residuals orthogonal to the explained part of Y: $\sum \hat{Y}_t \hat{u}_t = 0$
   Demo: $\sum(\hat{\alpha} + \hat{\beta} X_t)\hat{u}_t =$

$$\hat{\alpha} \underbrace{\sum \hat{u}_t}_{=0 \text{ (from prop 1)}} + \hat{\beta} \underbrace{\sum X_t \hat{u}_t}_{=0 \text{ (from prop 4)}} = 0$$

Causality: Y on X vs X on Y



Properties of residuals and SRF (5)

8. $\hat{\alpha}_{\text{OLS}}$ and $\hat{\beta}_{\text{OLS}}$ unbiased $\rightsquigarrow$ expected value = true value!
   Demo:

- $$\hat{\beta} = \frac{\sum y_t x_t}{\sum x_t^2}$$
  $$\mathsf{E}(\hat{\beta}) = \frac{1}{\sum x_t^2} \sum \underbrace{\mathsf{E}(y_t)}_{\beta x_t} x_t = \frac{1}{\sum x_t^2}\, \beta \sum x_t^2$$
  $$\boxed{\mathsf{E}(\hat{\beta}) = \beta}$$
- $$\hat{\alpha} = \overline{Y} - \hat{\beta}\overline{X}$$
  $$\mathsf{E}(\hat{\alpha}) = \frac{1}{T}\sum \mathsf{E}(Y_t) - \mathsf{E}(\hat{\beta})\overline{X}$$
  $$= \frac{1}{T}\sum(\alpha + \beta X_t) - \beta\overline{X} = \alpha + \beta\overline{X} - \beta\overline{X}$$
  $$\boxed{\mathsf{E}(\hat{\alpha}) = \alpha}$$

2.5a Goodness of Fit: the Coefficient of Determination (R^2).

Goodness of fit: Coefficient of determination

- Sum-of-Squares decomposition:

$$\sum Y_t^2 = \sum(\widehat{Y}_t^2 + \widehat{u}_t^2 + 2\widehat{Y}_t\widehat{u}_t)$$
$$= \sum \widehat{Y}_t^2 + \sum \widehat{u}_t^2 \qquad \text{(from prop 5)}$$

- $$\sum Y_t^2 - T\overline{Y}^2 = \sum \widehat{Y}_t^2 - T\overline{\widehat{Y}}^2 + \sum \widehat{u}_t^2 \qquad \text{(from prop 2)}$$
- $$\underset{(TSS)}{\sum y_t^2} = \underset{(ESS)}{\sum \widehat{y}_t^2} + \underset{(RSS)}{\sum \widehat{u}_t^2}$$
- Definition of R^2:

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$0 \leq R^2 \leq 1$ (Interpretation in terms of total variance??)

No intercept $\rightsquigarrow$ invalid R^2

SLRM: $Y_t = \beta X_t + u_t$,

- apply Least-Squares fit:

$$\min_{\beta} \sum_{t=1}^{T} u_t^2 \quad \text{where} \quad u_t = Y_t - \beta X_t :$$

- First derivatives:

$$\frac{\partial \sum u_t^2}{\partial \beta} = 2\sum u_t \frac{\partial u_t}{\partial \beta} = 2\sum u_t(-X_t)$$

- 1st.o.c. (minimum) $\Rightarrow$ first derivative $=$ zero:

$$\sum \widehat{u}_t X_t = \sum (Y_t X_t - \widehat{\beta} X_t^2) = 0$$

- $\nexists$ 1st equation!! $\rightsquigarrow$ $\begin{cases} \sum \widehat{u}_t \neq 0, \\ \overline{\widehat{Y}} \neq \overline{Y}, \end{cases}$ $\rightsquigarrow$ invalid R^2 (Why?)

Relationship of R^2 with correlation coef

$$R^2 = \frac{\frac{1}{T}\sum \widehat{y}_t^2}{\frac{1}{T}\sum y_t^2} = \frac{\frac{1}{T}\sum(\widehat{\beta}x_t)^2}{\frac{1}{T}\sum y_t^2} = \frac{\widehat{\beta}^2 \frac{1}{T}\sum x_t^2}{\frac{1}{T}\sum y_t^2}$$
$$= \widehat{\beta}^2 \frac{\text{Var}(X)}{\text{Var}(Y)} = \frac{\text{Cov}(Y,X)^2}{\text{Var}(X)^2} \frac{\text{Var}(X)}{\text{Var}(Y)}$$
$$= \frac{\text{Cov}(Y,X)^2}{\text{Var}(X)\,\text{Var}(Y)}$$

$R^2 = r_{X,Y}^2$

Numerical example: strawberry prod data (cont)

- recall data & previous calculations...
- do the same for fitted values...
- now calculate R^2...

	y^2	$\widehat{Y}$	$\widehat{y}$	$\widehat{y}^2$	$\widehat{u}$	$\widehat{u}^2$
	900	42.92	-27.07	732.82	-2.92	8.58
	100	53.08	-16.91	286.25	6.91	47.87
	400	63.23	-6.76	45.80	-13.23	175.09
	0	66.61	-3.38	11.45	3.38	11.45
	400	76.76	6.76	45.80	13.23	175.09
	100	90.30	20.30	412.21	-10.30	106.15
	900	97.07	27.07	732.82	2.92	8.58
Average	400	70	0	323.88		
Sum	2800			2267.17		532.82
	TSS			ESS		RSS

$R^2 = 0.8097$ $\left(= \frac{\text{ESS}}{\text{TSS}} = 1 - \frac{\text{RSS}}{\text{TSS}}\right)$

(Exercise: How does this compare with Corr(X,Y)? ... Try it!!)